Resonant switch model of twin peak HF QPOs applied to the source 4U 1636−53

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ABSTRACT

Resonant Switch (RS) model of twin peak high-frequency quasi-periodic oscillations (HF QPOs) assumes switch of twin oscillations at a resonant point where frequencies of the upper and lower oscillations ν_{U} and ν_{L} become commensurable and the twin oscillations change from one pair of the oscillating modes (corresponding to a specific model of HF QPOs) to some other pair due to non-linear resonant phenomena. The RS model is used to determine range of allowed values of spin a and mass M of the neutron star located at the atoll source 4U 1636−53 where two resonant points are observed at frequency ratios $\nu_{\mathrm{U}} : \nu_{\mathrm{L}} = 3:2, 5:4$. We consider the standard specific models of the twin oscillations based on the orbital and epicyclic geodetical frequencies. The resonant points are determined by the energy switch effect exhibited by the vanishing of the amplitude difference of the upper and lower oscillations. The predicted ranges of the neutron star parameters are strongly dependent on the twin modes applied in the RS model. We demonstrate that for some of the oscillatory modes used in the RS model the predicted parameters of the neutron star are unacceptable. Among acceptable RS models the most promising are those combining the Relativistic Precession and the Total Precession frequency relations or their modifications.

Keywords: *Stars: neutron — X-rays: binaries — Accretion, accretion disks*

1 Introduction

In the Galactic Low Mass X-Ray Binaries (LMXB) containing neutron (quark) stars quasiperiodic oscillations (*QPOs*) of X-ray brightness had been observed at low-(Hz) and high-(kHz) frequencies (see, e.g., van der Klis 2000, 2006; Barret et al. 2005). Since the high frequencies are close to the orbital frequency of the marginally stable circular orbit representing the inner edge of Keplerian disks orbiting neutron stars, the strong gravity effects must be relevant in explaining HF QPOs.

The HF QPOs in neutron star systems are often demonstrated as two simultaneously observed pairs of peaks (twin peaks) in the Fourier spectra corresponding to oscillations at the upper and lower frequencies ($\nu_{\mathrm{U}}, \nu_{\mathrm{L}}$). The twin peaks at the upper and lower frequencies ($\nu_{\mathrm{U}}, \nu_{\mathrm{L}}$) substantially change over time (in one observational sequence). Sometimes only one of the frequencies is observed and evolved (Belloni et al. 2007; Boutelier et al. 2010). Most of the twin HF QPOs in the so called

atoll sources (van der Klis 2006) have been detected at lower frequencies 600 – 800 Hz vs. upper frequencies 900 – 1200 Hz, demonstrating a clustering of the twin HF QPOs frequency ratios around 3 : 2. This clustering indicates some analogy to the black hole (BH) case where twin peaks with fixed pair of frequencies at the ratio 3 : 2 are usually observed (Abramowicz et al. 2003; Belloni et al. 2007; Török et al. 2008a,b; Boutelier et al. 2010). Amplitudes of twin HF QPOs in the neutron star systems are usually stronger and their coherence times are higher than those of BH sources (see, e.g., Remillard & McClintock 2006; Barret et al. 2005). It is probable that a 3 : 2 resonance plays a significant role also in the atoll sources containing neutron stars. However, this case is much more complicated, as the frequency ratio, although concentrated around 3 : 2, falls in a much wider range than in the BH systems (Török et al. 2005; Török 2009; Boutelier et al. 2010; Montero & Zanotti 2012).

In fact, a multi-peaked distribution in the frequency ratios has been observed, i.e., more than one resonance could be realized if a resonant mechanism is involved in generating the neutron star HF QPOs. For some atoll neutron star sources the upper and lower HF QPO frequencies can be traced along the whole observed range, but the probability to detect both QPOs simultaneously increases when the frequency ratio is close to ratio of small natural numbers, namely 3 : 2, 4 : 3, 5 : 4 – this has been observed in the case of six atoll sources: 4U 1636−53, 4U 1608−52, 4U 0614−09, 4U 1728−34, 4U 1820−30, 4U 1735−44 (Török 2009; Boutelier et al. 2010). The analysis of root-mean-squared-amplitude evolution in the group of six atoll sources shows that the upper and lower HF QPO amplitudes equal each other and alter their dominance while passing rational frequency ratios corresponding to the datapoints clustering (Török 2009). Such an "energy switch effect" can be well explained in a framework of non-linear resonant orbital models as shown in Horák et al. (2009). Moreover, the analysis of the twin peak HF QPO amplitudes in the atoll sources (4U 1636−53, 4U 1608−52, 4U 1820−30, and 4U 1735−44) indicates a cut-off at resonant radii corresponding to the frequency ratios 5 : 4 and 4 : 3 respectively implying a possibility that the accretion disk inner edge is located at the innermost resonant radius rather than at the innermost stable circular geodesic (ISCO) (Stuchlík et al. 2011).

The evolution of the lower and upper twin HF QPOs frequencies in the atoll sources suggests (a very rough) agreement of the data distribution with so called hot spot models, especially with the relativistic precession (RP) model prescribing the evolution of the upper frequency by $\nu_{\rm U} = \nu_{\rm K}$ and the lower frequency by $\nu_{\rm L} = \nu_{\rm K} - \nu_r$ (Stella & Vietri 1999, 1998). In rough agreement with the data are also other models based on the assumption of the oscillatory motion of hot spots, or accretion disk oscillations, with oscillatory frequencies given by the geodetical orbital and epicyclic motion: for example it holds for the modified RP1 model with $\nu_{\rm U} = \nu_\theta$ and $\nu_{\rm L} = \nu_{\rm K} - \nu_r$ (Bursa 2005), the total precession (TP) model with $\nu_{\rm U} = \nu_\theta$ (or $\nu_{\rm U} = \nu_{\rm K}$) and $\nu_{\rm L} = \nu_\theta - \nu_r$ (Stuchlík et al. 2007), the tidal disruption (TD) model with $\nu_{\rm U} = \nu_{\rm K} + \nu_r$ and $\nu_{\rm L} = \nu_{\rm K}$ (Čadež et al. 2008; Kostić et al. 2009), or the warped disk oscillations (WD) model with $\nu_{\rm U} = 2\nu_{\rm K} - \nu_r$ and $\nu_{\rm L} = 2\,(\nu_{\rm K} - \nu_r)$ (Kato 2004, 2008). In all of these models the frequency difference $\nu_{\rm U} - \nu_{\rm L}$ decreases with increase of the magnitude of the lower and upper frequencies in accord with trends given by the observational data (Belloni et al. 2007; Boutelier et al. 2010). This qualitative property of the observational data excludes the simple model of epicyclic oscillations with $\nu_{\rm U} = \nu_\theta$ and

$\nu_{\mathrm{L}} = \nu_r$ (Urbanec et al. 2010b) that works quite well in the case of HF QPOs in LMXB containing black holes (Török et al. 2005), or requires substantial modification of the model of epicyclic oscillations (Abramowicz et al. 2011).

The $\nu_{\mathrm{U}}/\nu_{\mathrm{L}}$ frequency relations, given by a variety of the relevant frequency-relation models mentioned above, can be fitted to the observational data for the atoll sources containing neutron (quark) stars, e.g., data determined for the atoll source 4U 1636−53 (Barret et al. 2005; Török et al. 2008a,b). The parameters of the neutron (quark) star spacetime can be then determined due to the fitting of the observational data by the frequency-relation models. The rotating neutron stars are described quite well by the Hartle–Thorne geometry (Hartle & Thorne 1968) characterized by three parameters: mass M, internal angular momentum J and quadrupole moment Q. It is convenient to use dimensionless parameters $a = J/M^2$ (spin) and $q = QM/J^2$ (dimensionless quadrupole moment). In the special case when $q \sim 1$, the Hartle–Thorne geometry reduces to the well known and well studied Kerr geometry that is convenient for calculations in strong gravitational field regime because of high simplicity of relevant formulae. Near-maximum-mass neutron (quark) star Hartle–Thorne models constructed for any given equation of state imply $q \sim 1$ and the Kerr geometry is applicable quite correctly in such situations instead of the Hartle–Thorne geometry (Török et al. 2010, Urbanec et al. in preparation). High neutron star masses can be expected in the LMXB as a result of accretion and in such a case the use of the simple Kerr geometry is justified.

Quality of the fitting procedure appears to be poor for the atoll source 4U 1636−53 showing resonant frequency ratios 3 : 2 and 5 : 4 (Török et al. 2012). Similar very bad fit of observational data to the frequency-relation models was found by Lin et al. (2011) for the atoll sources 4U 1636−53 and Sco X-1: the mass and spin determined by the fitting procedure are given with very large error for any of the applied models used in the paper, as also for some other models of the frequency relations (Miller et al. 1998; Osherovich & Titarchuk 1999; Zhang 2004; Zhang et al. 2006; Mukhopadhyay 2009; Shi & Li 2009; Chakrabarti et al. 2009; Shi 2011; Mukherjee & Bhattacharyya 2012). The strong disagreement of the data distribution and their fitting by the frequency-relation models based on the assumption of the geodesic character of the oscillatory frequencies related to proper combinations of the geodetical orbital and epicyclic motion caused attempts to find a correction of a non-geodesic origin reflecting some important physical ingredients – as e.g., influence of the magnetic field of the neutron star (Bakala et al. 2012, 2010; Kovář et al. 2008; Stuchlík & Kološ 2012), or of thickness of non-slender oscillating tori (Rezzolla et al. 2003; Straub & Šrámková 2009) that could make the fitting procedure much better, as shown in Török et al. (2012).

However, it is useful to consider another possibility to improve the fitting procedures that is not based on the non-geodesic corrections and modifies the present orbital resonance models based on the assumption of the geodesic orbital and epicyclic motion, or disk oscillations with frequencies related to those of the geodetical motion.

2 Resonant switch model of HF QPOs in neutron star systems

We propose a new model of twin peak HF QPOs assuming that the twin oscillatory modes creating sequences of the lower and upper HF QPOs can switch at a resonant point. According to such a Resonant Switch (RS) model non-linear resonant phenomena will cause excitation of a new oscillatory mode (or two new oscillatory modes) and vanishing of one of the previously acting modes (or both the previous modes), i.e., switching from one pair of the oscillatory modes to other pair of them that will be acting up to the following resonant point. We assume two resonant points at the disk radii r_{out} and r_{in}, with observed frequencies $\nu_{\mathrm{U}}^{\mathrm{out}}$, $\nu_{\mathrm{L}}^{\mathrm{out}}$ and $\nu_{\mathrm{U}}^{\mathrm{in}}$, $\nu_{\mathrm{L}}^{\mathrm{in}}$, being in commensurable ratios $p^{\mathrm{out}} = n^{\mathrm{out}} : m^{\mathrm{out}}$ and $p^{\mathrm{in}} = n^{\mathrm{in}} : m^{\mathrm{in}}$. These resonant frequencies are determined by the energy switch effect (Török 2009); observations put the restrictions $\nu_{\mathrm{U}}^{\mathrm{in}} > \nu_{\mathrm{U}}^{\mathrm{out}}$ and $p^{\mathrm{in}} < p^{\mathrm{out}}$. In the region covering the resonant point at r_{out} the twin oscillatory modes with the upper (lower) frequency are determined by the function $\nu_{\mathrm{U}}^{\mathrm{out}}(x; M, a)$ ($\nu_{\mathrm{L}}^{\mathrm{out}}(x; M, a)$). Near the inner resonant point at r_{in} different oscillatory modes given by the frequency functions $\nu_{\mathrm{U}}^{\mathrm{in}}(x; M, a)$ and $\nu_{\mathrm{L}}^{\mathrm{in}}(x; M, a)$ occur. We assume all the frequency functions to be determined by combinations of the orbital and epicyclic frequencies of the geodesic motion in the Kerr backgrounds. Such a simplification is correct with high precision for neutron (quark) stars with large masses, close to maximum allowed for a given equation of state. As demonstrated in Török et al. (2010) and Urbanec et al. (in preparation), the quadrupole moment of Hartle–Thorne geometry corresponding to near-extreme masses is very close to the Kerr limit and the orbital and epicyclic frequencies are very close to those given by the exact Kerr geometry.

The frequency functions have to meet the observationally given resonant frequencies. In the framework of the simple RS model, when two resonant points and two pairs of the frequency functions are assumed, this requirement enables determination of the parameters of the Kerr background describing with high precision the exterior of the neutron (quark) star. The "shooting" of the frequency functions to the resonant points, giving the neutron star parameters, can be realized efficiently in two steps. Independence of the frequency ratio on the mass parameter M implies the conditions

$$\nu_{\mathrm{U}}^{\mathrm{out}}(x; M, a) \quad : \quad \nu_{\mathrm{L}}^{\mathrm{out}}(x; M, a) = p^{\mathrm{out}}\,, \tag{1}$$

$$\nu_{\mathrm{U}}^{\mathrm{in}}(x; M, a) \quad : \quad \nu_{\mathrm{L}}^{\mathrm{in}}(x; M, a) = p^{\mathrm{in}} \tag{2}$$

giving relations for the spin a in terms of the dimensionless radius x and the resonant frequency ratio p. They can be expressed in the form $a^{\mathrm{out}}(x, p^{\mathrm{out}})$ and $a^{\mathrm{in}}(x, p^{\mathrm{in}})$, or in an inverse form $x^{\mathrm{out}}(a, p^{\mathrm{out}})$ and $x^{\mathrm{in}}(a, p^{\mathrm{in}})$. At the resonant radii the conditions

$$\nu_{\mathrm{U}}^{\mathrm{out}} = \nu_{\mathrm{U}}^{\mathrm{out}}(x; M, a)\,, \quad \nu_{\mathrm{U}}^{\mathrm{in}} = \nu_{\mathrm{U}}^{\mathrm{in}}(x; M, a) \tag{3}$$

are satisfied along the functions $M_{p_{\mathrm{out}}}^{\mathrm{out}}(a)$ and $M_{p_{\mathrm{in}}}^{\mathrm{in}}(a)$ which are obtained by using the functions $a^{\mathrm{out}}(x, p^{\mathrm{out}})$ and $a^{\mathrm{in}}(x, p^{\mathrm{in}})$. The parameters of the neutron (quark) star are then given by the condition

$$M_{p_{\mathrm{out}}}^{\mathrm{out}}(a) = M_{p_{\mathrm{in}}}^{\mathrm{in}}(a)\,. \tag{4}$$

The condition (4) determines M and a precisely, if the resonant frequencies are determined precisely. If an error occurs in determination of the resonant frequencies, as naturally expected, our method gives related intervals of acceptable values of mass and spin parameter of the neutron (quark) star.

Predictions of the RS model have to be tested. First, the predicted ranges of the mass and spin parameters have to be confronted with limits predicted by theoretical models of the neutron (quark) star structure. Second, these ranges have to be tested by the observational limits on mass and spin given by different phenomena observed in X-rays coming from the source, e.g., by the profiled spectral lines. Of course, crucial will be the test of fitting the twin peak HF QPO data around the resonant points with the results of the RS model.

Here we restrict ourselves to a rough confrontation of the neutron star theoretical predictions on the mass and spin. The other tests are under investigation and will be presented in a forthcoming paper.

3 Orbital and epicyclic frequencies of geodetical motion in the Kerr geometry

The formulae for the vertical epicyclic frequency ν_θ and the radial epicyclic frequency ν_r take in the Kerr spacetime (describing black holes or exterior of near-maximum mass neutron stars) the form (e.g., Aliev & Galtsov 1981; Kato et al. 1998; Stella & Vietri 1998; Török & Stuchlík 2005)

$$\nu_\theta^2 = \alpha_\theta\, \nu_{\mathrm{K}}^2, \qquad \nu_r^2 = \alpha_r\, \nu_{\mathrm{K}}^2, \tag{5}$$

where the Keplerian orbital frequency ν_{K} and the related epicyclic frequencies are given by the formulae

$$\nu_{\mathrm{K}} = \frac{1}{2\pi}\left(\frac{\mathrm{G}M}{r_{\mathrm{G}}^3}\right)^{1/2}\frac{1}{x^{3/2}+a} = \frac{1}{2\pi}\left(\frac{\mathrm{c}^3}{\mathrm{G}M}\right)\frac{1}{x^{3/2}+a}\,, \tag{6}$$

$$\alpha_\theta = 1-\frac{4\,a}{x^{3/2}}+\frac{3\,a^2}{x^2}\,, \tag{7}$$

$$\alpha_r = 1-\frac{6}{x}+\frac{8\,a}{x^{3/2}}-\frac{3\,a^2}{x^2}\,. \tag{8}$$

Here $x = r/(\mathrm{G}M/\mathrm{c}^2)$ is the dimensionless radius, expressed in terms of the gravitational radius.

The Keplerian frequency $\nu_{\mathrm{K}}(x, a)$ is a monotonically decreasing function of the radial coordinate for any value of the Kerr geometry spin. The radial epicyclic frequency has a global maximum for any Kerr black hole spacetime ($0 < a < 1$) and also the vertical epicyclic frequency is not monotonic if the spin is sufficiently high (see, e.g., Kato et al. 1998; Perez et al. 1997). For the Kerr black-hole spacetimes, the locations $\mathcal{R}_r(a)$, $\mathcal{R}_\theta(a)$ of maxima of the epicyclic frequencies ν_r, ν_θ are implicitly given by

the conditions (Török & Stuchlík 2005)

$$\beta_{\rm j}(x,a) \;=\; \frac{1}{2}\frac{\sqrt{x}}{x^{3/2}+a}\,\alpha_{\rm j}(x,a)\,, \quad \text{where } {\rm j}\in\{r,\theta\}\,, \tag{9}$$

$$\beta_r(x,a) \;\equiv\; \frac{1}{x^2}-\frac{2\,a}{x^{5/2}}+\frac{a^2}{x^3}\,, \tag{10}$$

$$\beta_\theta(x,a) \;\equiv\; \frac{a}{x^{5/2}}-\frac{a^2}{x^3}\,. \tag{11}$$

For any black hole spin, the extrema of the radial epicyclic frequency $\mathcal{R}_r\,(a)$ are located above the marginally stable orbit. The latitudinal epicyclic frequency has extrema $\mathcal{R}_\theta\,(a)$ located above the photon (marginally bound or marginally stable) circular orbit in the black hole spacetimes with spin $a > 0.748$ (0.852, 0.952) (Török & Stuchlík 2005). In the Keplerian disks with the inner boundary at $x_{\rm in} \sim x_{\rm ms}$, the limiting value $a = 0.952$ is relevant. Note that in the Kerr naked singularity spacetimes (with $a > 1$) the behaviour of the orbital and epicyclic frequencies is much more complex (Török & Stuchlík 2005; Stuchlík & Schee 2012) and also the related optical phenomena (as profiled lines) demonstrate strong differences in comparison to those created in the black hole or neutron star external spacetimes having $a < 1$ (Stuchlík & Schee 2010, 2012). However, these differences are important only at radii $r < M$, and become irrelevant at radii related to the exterior of the superspinning quark stars having (slightly) $a > 1$ (Lo & Lin 2011).

4 Frequency relations of the oscillatory mode pairs

We concentrate attention on the combinations of the relativistic precession (RP) and total precession (TP) frequency relations, defined by the frequency mix for the lower frequency, their modifications for the upper frequency, including the higher, "beat", harmonics, and the tidal disruption (TD) and warped disk (WD) models. For each of the frequency relations under consideration we present the function determining the resonant radii $x_{n:m}(a)$ given by the upper to lower frequency ratio $\nu_{\rm U} : \nu_{\rm L} = n : m$ characterized by the parameter

$$p = \left(\frac{m}{n}\right)^2\,. \tag{12}$$

Generally, the condition $x_{n:m} > x_{\rm ms}$ has to be satisfied; the marginally stable orbit $x_{\rm ms}$, assumed to correspond to the inner edge of the Keplerian disks, is implicitly determined by the function

$$a = a_{\rm ms}(x) \equiv \frac{\sqrt{x}}{3}\left(4-\sqrt{3x-2}\right). \tag{13}$$

4.1 Relativistic precession model

The RP model is determined by the frequency relation

$$\frac{\nu_{\rm K}}{\nu_{\rm K}-\nu_r} = \frac{n}{m}\,. \tag{14}$$

The resonance condition reads

$$a = a^{\mathrm{K}/(\mathrm{K}-r)}(x, p) \equiv \frac{\sqrt{x}}{3}\left[4 - \sqrt{3x(1 - p_{\mathrm{RP}}) - 2}\right] \tag{15}$$

where

$$p_{\mathrm{RP}} = \left(\frac{n-m}{n}\right)^2 = (1 - \sqrt{p})^2 \,. \tag{16}$$

Modification RP1. The upper frequency corresponds to the vertical epicyclic frequency, the modified frequency relation is determined by

$$\frac{\nu_\theta}{\nu_{\mathrm{K}} - \nu_r} = \frac{n}{m} \,. \tag{17}$$

The resonance function $a^{\theta/(\mathrm{K}-r)}(x, p)$ is properly chosen solution of the equation

$$p^2\alpha_\theta^2 - 2p\alpha_\theta\,(1 + \alpha_r) + (1 - \alpha_r)^2 = 0 \,. \tag{18}$$

We shall not give the function explicitly here because of its complexity.

Modification RPB. The upper frequency corresponds to the beat frequency, the modified frequency relation is determined by

$$\frac{\nu_{\mathrm{K}} + \nu_r}{\nu_{\mathrm{K}} - \nu_r} = \frac{n}{m} \,. \tag{19}$$

The resonance condition reads

$$a = a^{(\mathrm{K}+r)/(\mathrm{K}-r)}(x, p) \equiv \frac{\sqrt{x}}{3}\left[4 - \sqrt{3x(1 - p_{\mathrm{RPB}}) - 2}\right] \tag{20}$$

where

$$p_{\mathrm{RPB}} = \left(\frac{n-m}{n+m}\right)^2 = \left(\frac{\sqrt{p} - 1}{\sqrt{p} + 1}\right)^2 \,. \tag{21}$$

4.2 Total precession model

The TP model is determined by the frequency relation

$$\frac{\nu_\theta}{\nu_\theta - \nu_r} = \frac{n}{m} \,. \tag{22}$$

The resonance condition reads

$$\begin{aligned} a = a^{\theta/(\theta-r)}(x, p) \quad \equiv \quad & \frac{\sqrt{x}}{3(p_{\mathrm{TP}} + 1)}\Big\{2\,(p_{\mathrm{TP}} + 2) - \\ & - \sqrt{(1 - p_{\mathrm{TP}})\,[3x(p_{\mathrm{TP}} + 1) - 2(2p_{\mathrm{TP}} + 1)]}\Big\} \end{aligned} \tag{23}$$

where

$$p_{\mathrm{TP}} = \left(\frac{n-m}{n}\right)^2 = (1 - \sqrt{p})^2 \,. \tag{24}$$

Modification TP1. The upper frequency corresponds to the Keplerian orbital frequency, the modified frequency relation is determined by

$$\frac{\nu_{\mathrm{K}}}{\nu_\theta - \nu_r} = \frac{n}{m}\,. \tag{25}$$

The resonance function $a^{\mathrm{K}/(\theta-r)}(x,p)$ is solution of the equation

$$(\alpha_\theta - \alpha_r)^2 - 2p\,(\alpha_\theta + \alpha_r) + p^2 = 0\,. \tag{26}$$

In the explicit form the resonance condition reads

$$\begin{aligned} a &= a^{\mathrm{K}/(\theta-r)}(x,p) \equiv \\ &\equiv \sqrt{x} + \frac{1}{2\cdot 3^{5/6}} \times \left[\sqrt{\frac{A^{2/3}+B}{A^{1/3}}} - \sqrt{A^{1/3}\left(\frac{4\sqrt{3}px^{5/2}}{\sqrt{A+A^{1/3}B}} - 1\right) - \frac{B}{A^{1/3}}}\right] \end{aligned} \tag{27}$$

where

$$A = 6p^2x^5 + \sqrt{36p^4x^{10} - B^3}\,, \tag{28}$$

$$B = 3^{1/3}px^3\left[4 + (p-4)x\right]\,. \tag{29}$$

Modification TPB. The upper frequency corresponds to the beat frequency, the modified frequency relation is determined by

$$\frac{\nu_\theta + \nu_r}{\nu_\theta - \nu_r} = \frac{n}{m}\,. \tag{30}$$

The resonance condition reads

$$\begin{aligned} a = a^{(\theta+r)/(\theta-r)}(x,p) \equiv\ & \frac{\sqrt{x}}{3(p_{\mathrm{TPB}}+1)}\Big\{2(p_{\mathrm{TPB}}+2) - \\ & - \sqrt{(1-p_{\mathrm{TPB}})\left[3x(p_{\mathrm{TPB}}+1) - 2(2p_{\mathrm{TPB}}+1)\right]}\Big\} \end{aligned} \tag{31}$$

where

$$p_{\mathrm{TPB}} = \left(\frac{n-m}{n+m}\right)^2 = \left(\frac{\sqrt{p}-1}{\sqrt{p}+1}\right)^2. \tag{32}$$

4.3 Tidal disruption model

The TD model (Kostić et al. 2009) is based on the idea of an orbiting hot spot distorted by the influence of the tidal effects of the central black hole or neutron star. It is is determined by the frequency relation

$$\frac{\nu_{\mathrm{K}} + \nu_r}{\nu_{\mathrm{K}}} = \frac{n}{m}\,. \tag{33}$$

The resonance condition reads

$$a = a^{(\mathrm{K}+r)/\mathrm{K}}(x,p) \equiv \frac{\sqrt{x}}{3}\left[4 - \sqrt{3x(1-p_{\mathrm{TD}}) - 2}\right] \tag{34}$$

where

$$p_{\mathrm{TD}} = \left(\frac{n-m}{m}\right)^2 = \frac{\left(1-\sqrt{p}\right)^2}{p}\,. \tag{35}$$

4.4 Warped disk model

The warped disk oscillations (WD) model (Kato 2004, 2008; Wagoner 1999) assumes the frequency of the disk oscillations to be given by combinations of the (multiples) of the Keplerian and epicyclic frequencies. Usually, the inertial-acoustic and g-mode oscillations and their resonances could be relevant. We consider as an example the frequency relation

$$\frac{2\nu_{\mathrm{K}} - \nu_r}{2\,(\nu_{\mathrm{K}} - \nu_r)} = \frac{n}{m}\,. \tag{36}$$

The resonance condition reads

$$a = a^{(2\mathrm{K}-r)/(2\mathrm{K}-2r)}(x, p) \equiv \frac{\sqrt{x}}{3}\left[4 - \sqrt{3x\,(1-p_{\mathrm{WD}}) - 2}\right] \tag{37}$$

where

$$p_{\mathrm{WD}} = \left[\frac{2(n-m)}{2n-m}\right]^2 = \left[\frac{2\left(1-\sqrt{p}\right)}{2-\sqrt{p}}\right]^2. \tag{38}$$

5 Resonant switch model applied to the source 4U 1636−53

The observational X-ray data obtained for both the atoll and Z-sources indicate feasibility of the RS model at least in some of the observed sources. We shall test the atoll source 4U 1636−53, where the twin peak HF QPOs span a wide range of frequencies crossing the frequency ratios 3 : 2, 4 : 3 and finishing at 5 : 4, just near (or at) the inner edge of the accretion disk (Boutelier et al. 2010; Török 2009; Stuchlík et al. 2011). In the case of the 4U 1636−53 source, behaviour of the observed oscillations indicates presence of the resonant effects at disk radii where the frequencies are in the ratios 3 : 2 and 5 : 4 because of the energy switch effect occurring at these frequency ratios (Török 2009); therefore, the source is properly chosen for our test of the RS model.

5.1 The resonant frequencies

Using the results of Török (2009), the resonant frequencies determined by the energy switch effect are given in the outer resonant point with frequency ratio $\nu_{\mathrm{U}}/\nu_{\mathrm{L}} = 3/2$ by the frequency intervals

$$\begin{aligned} \nu_{\mathrm{U}}^{\mathrm{out}} &= \nu_{\mathrm{U0}}^{\mathrm{out}} \pm \Delta\nu^{\mathrm{out}} = (970 \pm 30)\,\mathrm{Hz}\,, \\ \nu_{\mathrm{L}}^{\mathrm{out}} &= \nu_{\mathrm{L0}}^{\mathrm{out}} \pm \Delta\nu^{\mathrm{out}} = (647 \pm 20)\,\mathrm{Hz}\,, \end{aligned} \tag{39}$$

while at the inner resonant point with frequency ratio $\nu_{\rm U}/\nu_{\rm L} = 5/4$ the frequency intervals are

$$\begin{aligned} \nu_{\rm U}^{\rm in} &= \nu_{\rm U0}^{\rm in} \pm \Delta\nu^{\rm in} = (1180 \pm 20)\,{\rm Hz}\,, \\ \nu_{\rm L}^{\rm in} &= \nu_{\rm L0}^{\rm in} \pm \Delta\nu^{\rm in} = (944 \pm 16)\,{\rm Hz}\,. \end{aligned} \tag{40}$$

Note that the resonant points determined by using the energy switch effect are in accord with observational data points crossing the lines of constant frequency ratios 3 : 2 and 5 : 4 as given in the standard papers (Barret et al. 2005; Belloni et al. 2007).

5.2 Theoretical limits on the mass and spin of neutron and quark stars

It is well known that the neutron star mass surely cannot exceed the critical value of $M \sim 3.2\,{\rm M}_\odot$ (Rhoades & Ruffini 1974). On the other hand, realistic equations of state put limit on the maximal mass of neutron stars around $M_{\rm maxN} \sim 2.8\,{\rm M}_\odot$ (Postnikov et al. 2010). In fact, the extremal maximum $M_{\rm maxN} \sim 2.8\,{\rm M}_\odot$ is predicted by the field theory (Müller & Serot 1996). The limit of $M_{\rm maxN} \sim 2.5\,{\rm M}_\odot$ is predicted by Dirac–Brueckner–Hartree–Fock approach in some special case (Müther et al. 1987) and the variational approaches (Akmal & Pandharipande 1997; Akmal et al. 1998) and other approaches (Urbanec et al. 2010a) allow for $M_{\rm maxN} \sim 2.25\,{\rm M}_\odot$. On the neutron star dimensionless spin the limit of $a < a_{\rm maxN} = 0.7$ has been recently reported, being independent of the equation of state (Lo & Lin 2011).

For the quark stars the maximal mass is expected somewhat smaller in comparison with the neutron stars because of softer equations of state assumed in modelling the quark stars, but masses around $M_{\rm maxQ} \sim 2\,{\rm M}_\odot$ are allowed (see, e.g., Glendenning 2000; Lo & Lin 2011). However, a substantial difference occurs in the limit on maximal spin, since even slightly superspinning states of quark stars with $a_{\rm maxQ} \geq 1$, exceeding the black hole limit, has been recently reported by Lo & Lin (2011), independently of the details of the equation of state for quark matter. Such a sharp difference between the limits on the maximal spin of neutron and quark stars could probably be explained by the fact that in quark (strange) stars the strong nuclear force helps in binding the stars (or could be the binding force in low mass strange stars), while no such effect can be present in the case of neutron stars.

In the Hartle–Thorne models of rotating neutron stars the spin of the star has to be related to its rotational frequency. The rotational frequency of the neutron star at the atoll source 4U 1636−53 has been observed at $f_{\rm rot} = 580\,{\rm Hz}$, or $f_{\rm rot} = 290\,{\rm Hz}$, if we observe doubled radiating structure (Strohmayer & Markwardt 2002). Of course, such a rotational frequency is substantially lower in comparison with the mass shedding frequency, and the Hartle–Thorne model can be applied quite well, predicting spins significantly lower than the maximally allowed spin.

The theory of neutron star structure then implies for a wide variety of realistic equations of state the spin in the range (Hartle & Thorne 1968, Urbanec et al. in preparation)

$$0.1 < a < 0.4\,. \tag{41}$$

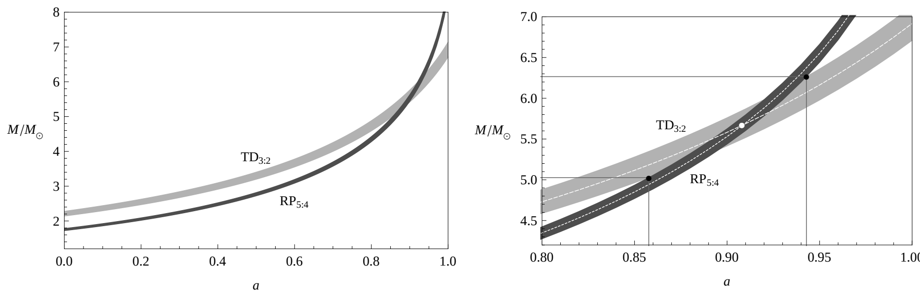

Fig. 1: *Left panel*: possible combinations of mass and spin of neutron star in the atoll source 4U 1636−53 predicted by the RP and TD model using the scatter of the resonant frequencies at each of the two resonant points where $\mathrm{RP} \equiv \nu_\mathrm{K} : (\nu_\mathrm{K} - \nu_r) = 5:4$ and $\mathrm{TD} \equiv (\nu_\mathrm{K} + \nu_r) : \nu_\mathrm{K} = 3:2$. The *right panel* gives the detailed information about the intervals of mass and spin relevant for this combination of the two frequency relations assuming the resonant switch model.

Of course, the upper part of the allowed spin range corresponds to the rotational frequency $f_\mathrm{rot} = 580\,\mathrm{Hz}$, while the lower part corresponds to $f_\mathrm{rot} = 290\,\mathrm{Hz}$. The related restriction on the neutron star (near-extreme) mass reads

$$M < 2.5\,\mathrm{M}_\odot\,. \tag{42}$$

Therefore, we shall use these restrictions and make a short comment on the results of the RS model in the case of three typical combinations considered in our study. We plan for a future work to make a detailed study considering Hartle–Thorne models constructed for a large variety of equations of state allowing for sufficiently large mass of the 4U 1636−53 neutron (or quark) star.[1]

5.3 Ranges of 4U 1636−53 neutron star mass and spin implied by the RS model

We have presented above a variety of the oscillatory mode pairs that deserve consideration and determined the representative resonance functions $a^{\nu_\mathrm{U}/\nu_\mathrm{L}}(x, p)$ related to the oscillatory pairs. Now we shall test all the possible combinations of the oscillatory pairs at the established resonant points and give the related intervals of allowed values of the mass and spin of the neutron or quark star. We shall discuss in detail two characteristic cases of the combinations RP–TD, RP–TP and its modification RP1–TP1. In the other cases, we shall give only the resulting allowed intervals of the neutron star parameters M and a. Of course, each pair of the frequency relations under consideration has to be properly ordered at the resonant points.

We have made the mass and spin estimates by "shooting" the frequency relations to the two resonant points for all the frequency relations presented above. The procedure

[1] Notice that quite recent study of the HF QPOs and line profiles observed in 4U 1636−53 leads the authors to claim that combining these two effects they are able to predict mass of the neutron star to be $\sim 2.4\,\mathrm{M}_\odot$ (Sanna et al. 2012).

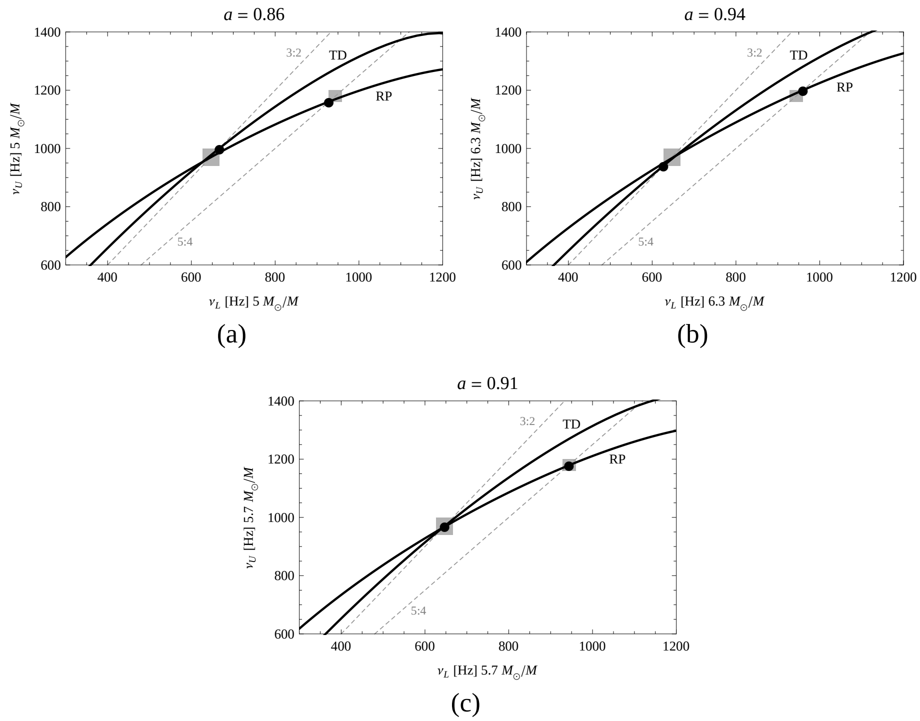

Fig. 2: RP and TD model fits for the atoll source 4U 1636−53. The gray rectangles indicate the scatter of the resonant frequencies at each of the two resonant points. The fits are shown for the minimal *(a)*, maximal *(b)* and central *(c)* allowed values of neutron star mass and spin implied by the resonant switch model. The case *(a)* corresponds to the left black point on the *right panel* of Fig. 1 with $\nu_{\mathrm{U}}^{\mathrm{out}}(3:2) = 1000\,\mathrm{Hz}$, $\nu_{\mathrm{L}}^{\mathrm{out}}(3:2) = 667\,\mathrm{Hz}$ and $\nu_{\mathrm{U}}^{\mathrm{in}}(5:4) = 1160\,\mathrm{Hz}$, $\nu_{\mathrm{L}}^{\mathrm{in}}(5:4) = 928\,\mathrm{Hz}$, *(b)* corresponds to the right black point on the *right panel* of Fig. 1 with $\nu_{\mathrm{U}}^{\mathrm{out}}(3:2) = 940\,\mathrm{Hz}$, $\nu_{\mathrm{L}}^{\mathrm{out}}(3:2) = 627\,\mathrm{Hz}$ and $\nu_{\mathrm{U}}^{\mathrm{in}}(5:4) = 1200\,\mathrm{Hz}$, $\nu_{\mathrm{L}}^{\mathrm{in}}(5:4) = 960\,\mathrm{Hz}$, and *(c)* corresponds to the central white point on the *right panel* of Fig. 1 with $\nu_{\mathrm{U}}^{\mathrm{out}}(3:2) = 970\,\mathrm{Hz}$, $\nu_{\mathrm{L}}^{\mathrm{out}}(3:2) = 647\,\mathrm{Hz}$ and $\nu_{\mathrm{U}}^{\mathrm{in}}(5:4) = 1180\,\mathrm{Hz}$, $\nu_{\mathrm{L}}^{\mathrm{in}}(5:4) = 944\,\mathrm{Hz}$.

of determining the intervals of mass and spin relevant for the combinations of the frequency relations is demonstrated in Figs. 1 – 4. The intervals of the mass and spin implied by the procedure of the RS model are summarized in Table 1. Only combinations giving a RS solution are presented in Table 1 where for the frequency relations under consideration the relevant resonant points are explicitly given.

The mass and spin estimates of the RS model related to the data of the 4U 1636-53 source have to be confronted with restrictions on the neutron star mass and spin implied by the theoretical models of neutron stars (or quark stars) and the observed rotational frequency of the neutron star at 4U 1636−53 presented above.

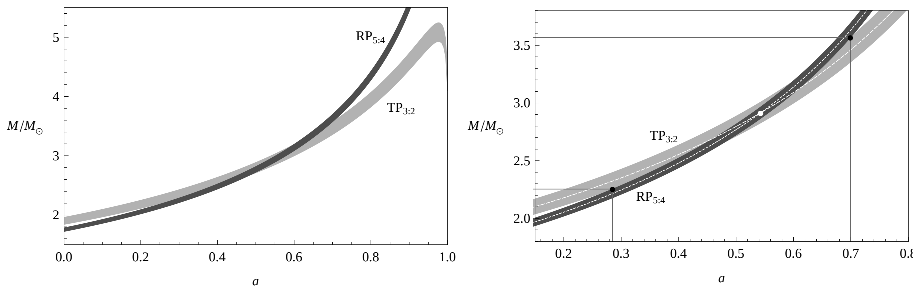

Fig. 3: *Left panel*: possible combinations of mass and spin of neutron star in the atoll source 4U 1636−53 predicted by the RP and TP model using the scatter of the resonant frequencies at each of the two resonant points where RP $\equiv \nu_{\rm K} : (\nu_{\rm K} - \nu_r) = 5:4$ and TP $\equiv \nu_\theta : (\nu_\theta - \nu_r) = 3:2$. The *right panel* gives the detailed information about the intervals of mass and spin relevant for this combination of the two frequency relations assuming the resonant switch model.

5.3.1 Combination of the RP and TD models

For the combination of RP and TD models, the RP model has to be related to the inner resonant point, while the TD model has to be related to the outer resonant point. We have demonstrated that for the frequency scatter at the resonant points, given by the Eqs. (39) and (40), the range of allowed values of the mass and spin of the neutron star is given by

$$0.86 < a < 0.94\,, \quad 5.03 < \frac{M}{\mathrm{M}_\odot} < 6.27\,. \tag{43}$$

The central point of the frequency ranges implies the central estimates of the neutron star parameters to be $a = 0.91$ and $M = 5.67\,\mathrm{M}_\odot$. For completeness, we present also the frequency dependence of the RP and TD oscillatory modes for the limiting values of the spin and mass and for the central values in Fig. 2. Qualitatively, these frequency dependencies could be in accord with the observational data.

Clearly, in the case of the combination of the RP and TD oscillatory modes the RS model implies parameters of the neutron star that are totally out of the ranges accepted by the recent theory of the structure of the neutron or quark stars both for their spin and mass. Therefore, the RP–TD combination can be excluded as a realistic explanation of the observed data in the atoll source 4U 1636−53 because of high values of the predicted neutron star parameters. Nevertheless, it is interesting to notice that the RP model fits in the allowed range of mass and spin both resonant points quite well, especially for the central point with $a = 0.91$ and $M = 5.7\,\mathrm{M}_\odot$, while the TD frequency relation is restricted just to the region corresponding to the outer resonant point.

5.3.2 Combination of the RP and TP models

For the combination of RP and TP models, the RP model has to be related to the inner resonant point, while the TP model has to be related to the outer resonant point. For the frequency scatter at the resonant points, given by the Eqs. (39) and (40), we deduce

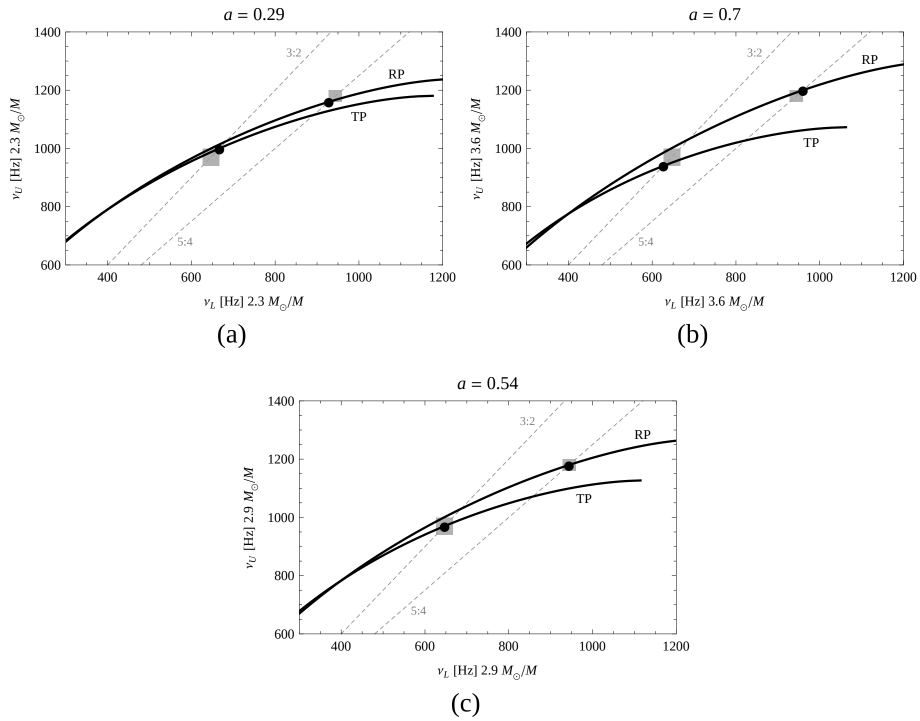

Fig. 4: RP and TP model fits for the atoll source 4U 1636−53. The gray rectangles indicate the scatter of the resonant frequencies at each of the two resonant points. The fits are shown for the minimal *(a)*, maximal *(b)* and central *(c)* allowed values of neutron star mass and spin implied by the resonant switch model. The case *(a)* corresponds to the left black point on the *right panel* of Fig. 3 with $\nu_{\mathrm{U}}^{\mathrm{out}}(3:2) = 1000\,\mathrm{Hz}$, $\nu_{\mathrm{L}}^{\mathrm{out}}(3:2) = 667\,\mathrm{Hz}$ and $\nu_{\mathrm{U}}^{\mathrm{in}}(5:4) = 1160\,\mathrm{Hz}$, $\nu_{\mathrm{L}}^{\mathrm{in}}(5:4) = 928\,\mathrm{Hz}$, *(b)* corresponds to the right black point on the *right panel* of Fig. 3 with $\nu_{\mathrm{U}}^{\mathrm{out}}(3:2) = 940\,\mathrm{Hz}$, $\nu_{\mathrm{L}}^{\mathrm{out}}(3:2) = 627\,\mathrm{Hz}$ and $\nu_{\mathrm{U}}^{\mathrm{in}}(5:4) = 1200\,\mathrm{Hz}$, $\nu_{\mathrm{L}}^{\mathrm{in}}(5:4) = 960\,\mathrm{Hz}$, and *(c)* corresponds to the central white point on the *right panel* of Fig. 3 with $\nu_{\mathrm{U}}^{\mathrm{out}}(3:2) = 970\,\mathrm{Hz}$, $\nu_{\mathrm{L}}^{\mathrm{out}}(3:2) = 647\,\mathrm{Hz}$ and $\nu_{\mathrm{U}}^{\mathrm{in}}(5:4) = 1180\,\mathrm{Hz}$, $\nu_{\mathrm{L}}^{\mathrm{in}}(5:4) = 944\,\mathrm{Hz}$.

the range of allowed values of the mass and spin of the neutron star given by

$$0.29 < a < 0.70\,, \quad 2.25 < \frac{M}{\mathrm{M}_{\odot}} < 3.57\,. \tag{44}$$

The central point of the frequency ranges implies the central estimates of the neutron star parameters to be $a = 0.54$ and $M = 2.91\,\mathrm{M}_{\odot}$. In Fig. 4 we present again the frequency dependence of the RP and TP oscillatory modes for the limiting values of the neutron star spin and mass and for the central values of these parameters. Again, these frequency dependencies are in accord with the observational data qualitatively. As in the case of TD–RP combination, the RP frequency relation meets both resonant points frequency intervals, but only slightly, while the TP relation meets only the outer resonant point frequency interval.

Table 1: Intervals of mass and spin of the neutron star in the atoll source 4U 1636−53 implied by the procedure of the RS model. The shaded rows represent those combinations of models that give acceptable values of spin and mass. Note that the model WD(3:2) gives identical solution as TD(3:2).

Combination of models	**spin** a	**mass** $[M/\mathrm{M}_\odot]$
RP(3:2) – RP(5:4)	0.68 – 0.97	3.60 – 6.88
RP(5:4) – RP1(3:2)	0.14 – 0.42	1.98 – 2.49
RP(5:4) – TP(3:2)	0.29 – 0.70	2.25 – 3.57
RP(3:2) – TP1(5:4)	0.27 – 0.74	2.28 – 4.20
RP(3:2) – TPB(5:4)	0.18 – 0.65	2.24 – 3.45
RP(5:4) – TPB(3:2)	0.90 – 0.94	5.63 – 6.27
RP1(3:2) – TP(5:4)	0.25 – 0.67	2.11 – 2.94
RP1(3:2) – TP(5:4)	0.988 – 0.994	3.64 – 3.85
RP1(3:2) – TP1(5:4)	0.10 – 0.34	1.94 – 2.36
RP1(3:2) – TPB(5:4)	0.992 – 0.996	3.65 – 3.87
RP1(5:4) – TPB(3:2)	0.9993 – 0.9996	3.47 – 3.59
RPB(5:4) – TP1(3:2)	0.18 – 0.67	2.30 – 4.11
RPB(5:4) – TPB(3:2)	0.72 – 0.84	4.52 – 5.56
TP(3:2) – TP1(5:4)	0.17 – 0.52	2.07 – 2.93
TP(3:2) – TPB(5:4)	0.48 – 0.94	2.84 – 4.71
TP1(3:2) – TPB(5:4)	0.08 – 0.38	2.10 – 2.70
TD(3:2) – TD(5:4)	0.00 – 0.69	2.14 – 4.22
TD(3:2) – RP(5:4)	0.86 – 0.94	5.03 – 6.27
TD(3:2) – RPB(5:4)	0.34 – 0.73	2.75 – 4.43
TD(3:2) – TP1(5:4)	0.70 – 0.74	3.99 – 4.33
TD(5:4) – TP1(3:2)	0.38 – 0.71	2.89 – 4.48
TD(5:4) – TPB(3:2)	0.70 – 0.85	4.38 – 5.58
WD(3:2) – RP(5:4)	0.86 – 0.94	5.03 – 6.27
WD(3:2) – RPB(5:4)	0.34 – 0.73	2.75 – 4.43
WD(5:4) – RPB(3:2)	0.00 – 0.86	2.56 – 7.02
WD(3:2) – TP1(5:4)	0.70 – 0.74	3.99 – 4.33
WD(5:4) – TP1(3:2)	0.85 – 0.89	6.64 – 7.59
WD(5:4) – TPB(3:2)	0.00 – 0.29	2.56 – 3.21
WD(3:2) – TD(5:4)	0.00 – 0.69	2.14 – 4.22

The RP–TP combination of the oscillatory modes in the RS model implies parameters of the neutron star that are quite acceptable in the lower end of the allowed ranges of spin and mass. Comparing our results with the allowed ranges of spin (Eq. 41) and

mass (Eq. 42) we can see that the RP–TP combination could work for $0.3 \lesssim a \lesssim 0.4$ and mass $2.25 \lesssim M/\mathrm{M}_\odot \lesssim 2.5$.

The physical explanation of the RP–TP model could be relatively very simple. A hot spot is oscillating in both vertical and radial directions and its radiation is modified by the frequencies ν_θ and $\nu_\theta - \nu_r$; approaching the $3:2$ resonant point, oscillations in the vertical direction are successively damped due to non-linear (e.g., tidal) effects and the radial oscillations are enforced. Then the Keplerian and the radial epicyclic frequencies become important.

5.3.3 Combination of the RP1 and TP1 models

Finally, we discuss one important case of combinations of modifications of the RP and TP frequency relations. We do not follow all the details, presenting only the results. The TP1 model has to be related to the inner resonant point, while the RP1 model has to be related to the outer resonant point. For the frequency scatter at the resonant points, given by the Eqs. (39) and (40), we deduce the range of allowed values of the mass and spin of the neutron star given by

$$0.10 < a < 0.34\,, \quad 1.94 < \frac{M}{\mathrm{M}_\odot} < 2.36\,. \tag{45}$$

The central point of the frequency ranges implies the central estimates of the neutron star parameters to be $a = 0.23$ and $M = 2.15\,\mathrm{M}_\odot$.

The RP1–TP1 combination of the oscillatory modes in the RS model implies complete ranges of parameters of the neutron star that are in a good accord with the theoretical limits on the spin (Eq. 41) and mass (Eq. 42) of the neutron star in 4U 1636-53. This combination could thus be considered as one of the best candidates for explaining the observed data of the HF QPOs.

We can see from Table 1 where all the results are presented that some combinations of the frequency relations are clearly excluded by the theoretical limits on mass and spin of neutron stars, while other give quite reasonable restrictions on the mass and spin parameters of the neutron star present in the atoll source 4U 1636−53. We plan to test further the acceptable combinations of the frequency relations by fitting the relations to the observational data related to the vicinity of the resonant points. Our preliminary results indicate that the improvement of the fitting procedure precision could be really relevant. We believe that than we are able to fix more precisely the proper combination of the frequency relations.

6 Conclusions

The RS model has been tested for the atoll source 4U 1636−53 demonstrating possible two resonant points in the observed data. For relevant pairs of the oscillatory frequency relations the range of allowed values of the mass and dimensionless spin of the neutron star are determined giving in some cases acceptable pairs of frequency relations, while some other pairs are excluded because of predicting unacceptable values of the spin and/or mass of the neutron star at 4U 1636−53. We focused our attention on the test of

the frequency relations containing geodetical orbital and epicyclic frequencies or some combinations of these frequencies. It should be noted that the cause of the switch of the pairs of the oscillatory modes is not necessarily tied to the resonant phenomena related to the oscillations governed by the frequencies of the geodetical motion, as the switch can be related, e.g., to the influence of the magnetic field of the neutron star, or the radiation coming from the surface of the neutron star. Moreover, in some sources, e.g., the Circinus X-1, we cannot exclude the possibility of chaotic changes of frequency relation pairs in a given fixed interval of observed frequencies. We study the resonant phenomena first, leaving other cases to the future studies.

Generally, the RP–TP combination of the RS model, and its modifications, enable acceptable explanation of the observational data for 4U 1636−53 source. This should be, however, explicitly tested by fitting procedure applied to the observed twin HF QPO sequences related to the resonant points. We plan to make such a test in a future work and also to test the RS model in the case of some other atoll (4U 1608−52) or Z (Circinus X-1) sources, containing a neutron (quark) star, with observational data indicating possible existence of two resonant points, and to estimate allowed values of the spin and mass of the neutron (quark) stars.

In the special situations related to accreting neutron stars with near-maximum masses the Kerr metric can be well applied in calculating the orbital and epicyclic geodetical frequencies, as has been done in the present paper, where the results of the mass and spin interval findings are in agreement with the assumption of near-maximum masses of the neutron stars. In general situations, when the neutron star mass is not close to its maximum value allowed by the equation of state, or when a precise fitting procedure of the observational data is needed, the Hartle–Thorne geometry describing rotating neutron stars has to be considered and the orbital and epicyclic frequencies reflecting influence of mass, spin and quadrupole moment of the neutron star has to be used. Nevertheless, the role of the quadrupole moment is relevant only very close to the inner edge of the accretion disk (Török et al. 2010).

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